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Gravitational Optics in Curved Spacetimes: Contrasting Newtonian and Relativistic Perspectives

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Dedicated to Fanny

Preface

It has been five years since the first edition has appeared. In that time, the book made it to third place in the BookAuthority's list of the all time 100 best books on General Relativity with a 4.14 average. It remained their until one of their "authorities" actually read the book, and it was quickly removed from the list. Among their primary collaborators is Elon Musk. That says it all! The book also made it to the Black Holes list, and, again every trace of it was removed when it was realized that the book is anti-general relativity and anti-what conventional wisdom tells us about the existence of black holes.

I thought about a second edition of the book when I realized an error in Chapter 2, Eqn (2.5) in that the Newtonian slope is either a straight line or a circle, depending on how the trajectory is traversed. It, in fact, makes the radius of curvature, (2.1), infinite! It is anyone's guess why this error was never picked up in the almost infinite commentaries on How Newton would have proceeded with his proof of the derivation of the elliptical orbit given his inverse square law for the central force. The error I bear responsibility for is in the transition from Eqn (2.6) to Eqn (2.8): Dots cannot replace primes, i.e., the derivative with respect to the angle variable cannot simply be replaced by the time derivative. Thus, came to be the writing of a second edition.

The first thing was to correct "Newton's slope" $r'/r = \pm\tan\theta$, where θ is the complementary angle that the tangent to the curve makes with the trajectory of the curve. Then the question arises as to why all the proofs of the derivation of the ellipse go through two integrations of the denominator of the radius of curvature when it is set equal to a constant. This constant supposedly incorporates the fact that the central force is inverse-square. Why not just integrate the equation of Newton's slope.

One of the arbitrary integration constants in the derivation of the equation of an ellipse is not so arbitrary for it represents the existence of a central potential. In its absence, the ellipse degenerates to a straight line as it should. Then a comparison with Weber's force brings up the interesting point that whereas Weber's force takes into account the orientation (angular dependency) of two relative charges in motion, it does not take into account their relative rotational energies. Once this is accounted got, Weber's force is seen to consist of the relative contraction of the particles in the direction of motion, the radial acceleration, and the angular momentum, just like in Newton's equation for the force. But, unlike Newton's mechanical equation, Weber's equation introduces a limiting speed–more than half a century before relativity was born.

Generalizations to Newton's slope arise when the metric coefficient of the angular term in the metric is no longer proportional to the area of a circle. A major distinction between elliptic and hyperbolic planes of constant curvature lies in the fact that the latter can support both open and closed trajectories. Considering the latter, we find the equation of a soliton, or the equation of a Joukowski ellipse. Apart from the technical area of the aerodynamics of wing foils, the Joukowski transform rose to prominence in the transformation of a circle to an ellipse in the complex plane. The resulting ellipse has its center at the origin and has a Hookean central force. The square of the Joukowski ellipse is another ellipse that displaces the origin to the focus of the ellipse and with it the transition from the Hookean law to a Newtonian inverse square.

We will discuss these dual laws henceforth. But in the new chapter on curvature, we obtain a non-traditional form of a conic independent of any constant of integration. The nonlinearity of the hyperbolic plane has brought in particle like solutions called solitons that propagate undistortedly on the surface of constant negative curvature called a pseudosphere. The Binet equation implicates a central force of inverse-fifth, which is the only self-dual law that exists at small radial distances, while distortions occur at greater distances.

We use Newton's impact method to derive the force laws both in the elliptic and hyperbolic planes of constant curvature. There is no simple prescription that allows us to replace the radial distance in the Euclidean plane with the Lobachevskian ”straight” line in the hyperbolic plane.

Another property will we investigate is the appearance of cusps in the non-Euclidean planes of constant curvature. Cusps in otherwise continuous trajectories first appeared in epicycles where a close trajectory orbits a larger closed trajectory. Cusp also appear in Euclidean trajectories, notably the cardioid which has a heart-shaped trajectory. When the force is attractive, the Binet equation has the form of Einstein's modification for the deflection of light. However, we would expect such a trajectory to be open, and not closed, and this will will discuss in due course.

If we allow for nonlinearities in the metric coefficient involving translation, we get generalized Ampère's laws and surfaces of revolution. Whereas Weber's force does not lead to a hyperbolic surface of constant curvature, its generalization does. Again the Joukowski ellipse appears in conjunction with the pseudosphere, and the possibility of particle like-solutions called solitons. Particle solutions always appear when there is more than one stationary solution since no stationary can be globally stable.

In fact, gravitational force have long been noted to be associated with the pseudosphere. It has been known since Newton's time that a doubly connected bugle can defy gravity when it ascends a ramp at large inclination angles. Also these amazing properties were documented as early as 1694, the above list of phenomena are new additions, some of which are still not clearly understood, that we will touch upon in this second edition of *Seeing Gravity*.

This only illustrates the adage that it is only through mistakes that new discoveries are made.

Pervolia, August 2024 Bernard Lavenda

Preface to the first edition

The first question that might come to mind when the reader picks up a copy of this book is: Why write a book on the optical properties of gravity? Optics is what you see, whereas gravity is what you feel. Newton kept them distinct, dedicating *Principia* to the laws of physics and gravitation, and *Opticks* dealing with a "Treatise on Reflexions, Inflexions, and Colours of Light."

Curiously enough, Einstein‘s first attempt to explain the deflection of light by the Sun employed Snell's law (and not Huygens‘ principle as he assumed) where he claimed that

> The principle of the constancy of the velocity of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity.

This would mean that a space varying gravitational field will decrease the speed of light in its neighborhood just like a varying index of refraction. Whereas in his subsequent formulation of the General theory of Relativity (GR) he assumed, following Poincarè, that gravity like light travels at the speed of light. And like light which is propagated as electromagnetic waves (EM), gravity should also be propagated as waves, Gravitational Waves (GWs).

Specifying that all particles follow geodesics made it equivalent to geometrical optics, which saw him derive a modified equation of a Keplerian orbit, in which he obtained numerical correspondence both with perihelion advance of Mercury and the deflection of light by the Sun in one fell swoop.

This seems rather incredible that the same law should hold good for a heavenly body as for a ray of light, by only neglecting a single term in that equation. In fact, it is the optical analogy to gravitation that is the thread that binds Newton‘s theory of gravitation with Einstein‘s General Relativity. Moreover, there is no reason to stop at Einstein‘s modification of the equation of a Keplerian orbit, thus opening up a whole range of possibilities from Kepler‘s conics to Cassini ovals, and beyond.

A new branch of optics, dealing with dielectric materials, known as metamaterials, has benefited greatly from the analogy with relativistic mechanics. New phenomena, such cloaking and perfect imaging, are consequences of the fact that when light rays are focused by a lens they do not conform to Euclid‘s fifth postulate, but, rather, display a vast range of non-Euclidean behavior: Light perceives a medium as a curved space, and, so too, gravity perceives a medium of curved spacetime.

In all fairness, the idea is not new but can be traced back to, of all people, Maxwell who, in 1854 gave an example of an absolute instrument using spherical geometry.

Although Hooke and Newton were personal adversaries, Hooke‘s linear law and Newton‘s inverse square law are duals to one another, in the exact same way that a Luneburg lens is the dual of an Eaton lens. Moreover, Luneburg showed that ellipses in the plane were stereographic projections of a perfect optical instrument on a sphere, which is none other than Maxwell‘s example of an absolute instrument. So already optics has been given generalizations of Keplerian orbits when we 'lift our eyes to the heavens,' where light can travel in circular orbits. Is there a corresponding analogue for gravitation? Is the theory limited to conical sections, or are there a generalizations of orbits obtained from toridal sections, since a torus is also part of a set of Riemann surfaces?

Like optical and gravitational analogies, there must be electromagnetic and gravitational analogues based on the very nature of light. When Ampère came on the scene at the dawn of the nineteenth century people were concerned about the otherwise instantaneous propagation of the Coulomb force, which was resolved by Weber in his law of force of moving and static charges. Although many astronomers tried to apply the same equation to gravitation there was no common consensus, and Laplace's argument that gravity propagates some 7 million times faster than the speed of light could hardly be refuted. Any finite speed of propagation of the gravitational force would mean that it would display diffraction phenomena, and, in particular, aberration that would deflect the action of an otherwise radial force.

An illustration of the union of the EM and GW was found by a German school teacher named Paul Gerber just before the turn of the twentieth century, who was able to get the correct advance of the perihelion of Mercury. However, no one realized that his modification of the equation of the orbit was precisely the Weber force of electrodynamics, and it moreover predicted that gravity travels at $1/\sqrt{3}$ the speed of light. This was later confirmed by Schroedinger, over a quarter of a century later, in his attempt to by-pass GR using only classical mechanics. Whereas Schroedinger knew the value of the parameter in advance, Gerber's matching with Weber's force would have constrained it to be that value, and none other. This would no longer have been considered ‘gap’ fitting.

After developing these analogies it will bring me to the comparison of EM waves and GWs. Both supposedly propagate at the same finite speed, yet the former can propagate in the vacuum while the latter is viewed as ‘ripples’ of space-time, Einstein‘s new ether. EM waves can be shielded; GWs cannot. EM waves manifest diffraction phenomena; none is known for GWs. The energy of EM waves can be localized; GWs cannot. Both are considered transverse waves with two states of polarization.

And here is where I can offer some novelty and reserve. I give what can be considered a one line derivation of the Peter-Mathews expression for the decrease of the period of a binary that they derived from a laborious procedure in GR, and show the luminosity to be four orders of magnitude higher in aberration than that of black-body radiation. That translates into an unheard of Stefan law of T^7, compared to black body which has a T^4 law where T is the absolute temperature. And it predicts that GWs have the same number of degrees-of-freedom as GR without having to introduce pseudo-tensors, or worry how a Poynting vector can be employed when gravitational energy cannot even be localized. All this would tend to imply that GWs are much closer to their EM wave cousins than would have been expected.

In the journey I take in this book I have been influenced by relatively few people with the notable exceptions of the late Tom Van Flandern, whose physical intuition I concur with, Angelo Loinger, whose insistence on the ”purity” and limitations imposed by the founding fathers of GR should not be messed with, and to my late friend, Fred Cooperstock, who believed that ”Electromagnetic waves have an intrinsic duality; they are necessarily also gravitational waves."

This book was (and maybe still is) under contract for publication with World Scientific Publishing Company. After having submitted the manuscript, the in-house editor decided to send it out for review–a procedure that should have been carried out *prior* to the stipulation of the contract. After several months of delay, five reviewer comments came back without a single specific criticism–just a common feeling of 'concern.' The concern was over my criticism of the 'discovery' of gravitational waves, which prompted the in-house editor to 'suggest a major revision' wherein all 'unwarranted' criticisms of general relativity should be removed. This was nothing less than a 'gagging' order–something unbecoming of a publisher who claims to be 'neutral.' The fear of readership decline is enough to suppress any and all criticisms of a theory that has been pushed beyond its limits. The publisher should have thought of that prior to the publication of my other two books: *A New Perspective on Relativity: An Odyssey in Non-Euclidean Geometries* and *Where Physics Went Wrong*, which are along the same lines. It is fairly safe to say that Einstein, would he return today, would not recognize his own theory, and, moreover, would be appalled by some of the results that its numerical 'extension' has obtained that contradict the basic premises of his theory.

Pervolia & Shoresh, April 2019 Bernard Laver

Introduction

0.1 The present state of the art

A half a century ago, General Relativity (GR) was a side-show of a side-show. Even Einstein didn‘t think of his theory as anything more than rounding off Newtonian theory in being able to predict minute effects like the advance of the perihelion of Mercury. In a somewhat apologetic mood, Einstein wrote in his Forward to Bergmann‘s book: [1]

> It is true that the theory of relativity, particularly the general theory, has played a rather modest role in the correlation of empirical facts so far. . . It is quite possible, however, that some of the results of the general theory of relativity, such as the general covariance of the laws of nature and their nonlinearity, may help. . .

How the times have changed!

What Einstein was referring to was quantum theory which hogged center stage for the remainder of Einstein's life. Controversy still surrounded the existence of GWs In the 1957 Chapel Hill conference, Feynman was successful in convincing the majority of participants that GWs do, in fact, exist. He did so by using a 'sticky bead' analogy that assumed *a priori* that GWs carry energy which could be transferred to the sticky beads that would show up in frictional heat. In those days, physics was done by voting.

[1] P G Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Englewood Cliffs NJ, 1942)

Yet, the energy stress tensor in Einstein's field equations contains all forms of energy– except gravitational energy. It turns out that this energy is supposed to be accounted for by a gravitational pseudo-tenors, which being composed of the metric and its first derivatives does not contain the necessary second derivatives that would make it a tensor.

Pseudo-tensors, like the Christoffel connection coefficients of which they are composed, can be made to vanish by a mere change of coordinates. They can even appear in Euclidean flat space through a choice of coordinates, for example, cylindrical coordinates. So even if one is willing to accept the existence of GWs, they must be very different than how GR realizes them.

GWs are ripples in spacetime, and unlike electromagnetic (EM) waves, they need a medium to propagate in, yet they travel at the same speed as light. But, maybe the GWs are not the same as the gravitational force, and whereas the former propagate at the speed of light, the latter propagates instantaneously.

This would be analogous to the Coulomb field acting instantaneously while the EM fields propagate at the speed of light. But when Coulomb proposed his law in strict analogy with Newton's inverse-square law, the connection between electricity and light was unknown.

In Weber‘s (Wilhelm not Joseph) theory of EM both static and motional fields found coexistence through the introduction of a constant as a limiting speed of propagation of electric charges. The same approach appears in the Lorentz force which considers the sum of a static Coulomb potential and a motional magnetic field. Allowing the constant to tend to infinity would reproduce Coulomb's law.

Is there an analogous expression between Newton's law, and what would be the gravitational analogue of the Grassmann force? Although this would support the nature of the supposed polarization of GWs, it would introduce a lot of other problems because GWs wouldn't manifest the same optical characteristics as light, once beyond the geometric optical limit of small wavelength. It would necessary implicate an orthogonal field to the gravitational field, which has been called a gravito-magnetic force in analogy with the magnetic field.

The explanation given by LIGO of what causes GWs, like the collision of binary black holes, is based on Numerical Relativity (NR) which is a nascent field that uses computers to solve Einstein's equations numerically from prescribed initial data. For example their exist numerical codes that suggest a collapsing star emits between 1 and 2 % of its total mass in the form of GWs.

Which part of Einsteins equations contain energy dissipation and radiation is left unspecified. All this is very surprising in view of the fact that GR can't even solve the two-body problem. And what is even more remarkable that it confirms that two colliding impulsive GWs produce a singularity.

The procedure [2] is to discretize the Einstein field equations where arbitrary source functions are introduced that supposedly "encode the gauge freedom of the solution." Black holes are already built into the system by using a "scalar field gravitational collapse to

[2]F Pretorius, "Evolution of binary black hole spacetimes," arXiv:gr-qc/050714.

1 Curvature: The Geometry of Force

1.1 The Radius of Curvature

When Newton came on to the world stage during the second half of the seventeenth century, the table had been prepared for him by Kepler during the first half. Specifically, Kepler laid down his three laws governing celestial motion, and the time was ripe to determine the force, or forces, involved in guiding the planets in their orbits about the Sun.

It was common in those days to divide the problem into two: The 'direct' and 'indirect' problems. In the former, the path was given together with a center of force, and the aim was to determine the force necessary to maintain that orbit. This was a natural for Newton for he already knew that the orbit was elliptical.

In the indirect problem we are given the force and its center of action, and seek to determine the orbit. This is the problem of actual interest because we have universally embraced Newton's law of gravity as being an inverse square law.

To arrive at this conclusion, Newton concerned himself with curvature: Curvature should be an imprint of force. In the early 1670s, Newton derived an expression for the radius of curvature, ϱ, in polar coordinates (r, ϑ):

$$\varrho = \frac{r(1+z^2)^{3/2}}{1+z^2-z'}, \tag{1.1}$$

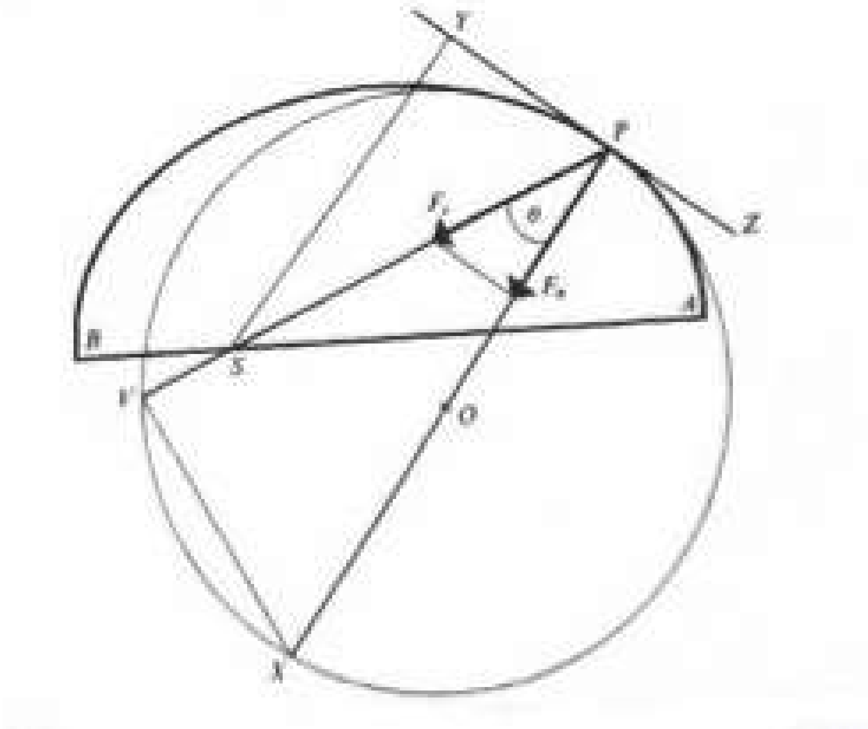

Figure 1.1: Newton‘s revised diagram for his fundamental theorem for the second and third editions of the *Principia* with the osculatinq circle inserted. Taken from J Bruce Brackenridge, "The critical role of curvature in Newton‘s developing dynamics," in *The Investigation of Difficult Things* P M Harman, A E Shapiro, eds (Cambridge U P, Cambridge, 1992) p 231-260.

where $z = (1/r)r'$ is the slope of the curve, and the prime stands for differentiation with respect to ϑ. We will now see how the denominator in (1.1) determines the law of force.

In order to do so, Newton used two additional relations requiring only that the force be directed toward its source. This limits the motion to a plane. Figure (1.1) appeared in Proposition 6, Theorem 5 of the revised edition of the *Principia.*

The trajectory APB has a tangent ZPY which is normal to the line PS, where S is the center of force. PVX is the circle of curvature whose radius ϱ is OP.

Newton‘s Proposition 1 contains Kepler‘s area law,

$$L = vr\sin\alpha, \tag{1.2}$$

where the instantaneous radius is SP, and v is the tangential velocity at the point P. All central forces conserve the angular momentum L, which is proportional to the area swept out by the satellite per unit time. The angle α is formed from the tangent ZPY and the radius SP. It‘s complementary angle ϑ is shown in the figure.

The second relation used by Newton is the force that would result if the actual trajectory were to be replaced by the osculating circle of curvature whose radius is ϱ. The centripetal force, F_0 is related to the central force F_c according to

$$F_c = F_0\cos\vartheta = F_0\sin\alpha = \frac{v^2}{\varrho}, \tag{1.3}$$

in accordance with Proposition 4. Implied throughout is Newton‘s assumption of *uniform circular motion*, i.e., constant angular speed as well as centripetal acceleration.

Introducing Kepler‘s areal law (1.2) into the expression for centripetal acceleration, (1.4), leads to the expression

$$F_c = \frac{L^2}{r^2\varrho\sin^3\alpha} \tag{1.4}$$

for the central force. Newton‘s proof would be complete if he could show that

$$\varrho \sin^3 \alpha = \text{const.}$$

In terms of the complementary angle, α, the slope of the trajectory is

$$z = \cot \alpha \tag{1.5}$$

so that

$$1 + z^2 = 1/\sin^2 \alpha,$$

which appears in the numerator of the expression for the radius of curvature, (1.1). Introducing that expression into the central force results in

$$\begin{aligned} F_c &= \frac{L^2}{r^3}\left(1 + 2\frac{r'^2}{r^2} - \frac{r''}{r}\right) \\ &= \frac{L^2}{r^3}\left(1 + z^2 - \frac{r}{2}\frac{d}{dr}(1+z^2)\right). \end{aligned} \tag{1.6}$$

In terms of the angle θ Newton’s slope (1.5) is

$$z = \frac{1}{r}\frac{dr}{d\theta} = \frac{r'}{r} = \tan\theta. \tag{1.7}$$

Introducing the inverse radial coordinate, $u = 1/r$(1.6 transforms into the Binet equation

$$u'' + u = \frac{F_c}{L^2u^2} \tag{1.8}$$

In order that the right-hand side be a constant, the central force must be inverse-square, $\propto 1/r^2$.

Rather than dealing with the Binet equation directly (1.8) authors like J Bruce Brackenridge deal with the curvature equation

$$\frac{d}{dr}(1+z^2) - 2(1+z^2)/r = -\frac{2}{L^2}(F_c r^2). \tag{1.9}$$

Since the Binet equation establishes that the central force is inverse-square, we can set the right-hand side equal to a constant, $2A$; then letting $f = 1 + z^2$, (1.9) becomes the ordinary inhomogeneous differential equation

$$\frac{df}{dr} - 2f/r = -2A.$$

The complementary solution, obtained by setting the right-hand side equal to zero is $f_c = Cr^2$, where C is an arbitrary constant of integration. Since the particular solution is $f_p = 2Ar$, the complete solution is the sum of the two, viz.,

$$f = f_c + f_p = 2Ar + Cr^2.$$

Reintroducing $1 + z^2$ for f, and writing $B^2 - A^2$ for C, where B is another in arbitrary constant of integration, allows Newton's slope to be written as

$$z = \frac{r'}{r} = r\sqrt{B^2 - \left(\frac{1}{r} - A^2\right)}. \tag{1.10}$$

Integration leads to

$$\theta = \int \frac{dr}{r^2\sqrt{B^2 - (1/r - A)^2}} = \cos^{-1}\frac{1/r - A}{B} - \alpha,$$

where α is another arbitrary constant of integration. Rearranging, we come out with the equation of an ellipse,

$$\frac{1}{r} = A + B\cos(\theta + \alpha). \tag{1.11}$$

From this Brackenridge [1] concludes

> Thus, as Newton has claimed, given the curvature from the force, the path is uniquely determined. Whether Newton could have produced a version of this proof, as Whiteside claims, is a matter of personal conviction.I, for one, have no doubt that he could. But he need not have done so, for the outline provided in Corollary 1 of Proposition 13 is adequate.

Thank heavens that Newton didn't present such a proof! This is nothing in Proposition 13 Corollary 1 regarding his slope (1.7). The only place mentioned is in conjunction of the logarithmic spiral where it is constant.

For the question immediately arises why perform two integrations, introducing two arbitrary constants when one would suffice by simply integrating Newton's slope, (1.7)? Actually, there should be a $\pm$ in that equation to take into account the ambiguity in how the path is traversed. Choosing the positive sign, a simple integration equation results in

$$\ln r = -\ln\cos\theta - \ln B, \tag{1.12}$$

where B is an arbitrary constant of integration. Equation (1.12) is

[1] J B Brackenrdige, "The critical role of curvature in Newton's developing dynamics, " in *The investigation of difficult things*, eds P. M. Harman & A. C. Shapiro, Cambridge U. P., 1992, p. 231.

an equation of a straight line! It is what you get when you set $A = 0$ in (1.11)! The negative sign gives a circle passing through the origin. And we know that the force responsible for that is an inverse-fifth power of the radial coordinate [2]

The Newtonian force, (1.6),

$$F_c = r\dot{\theta}^2 - \ddot{r}, \tag{1.13}$$

where the dot indicates differentiation with respect to time is not comparable to Weber's law between charges e and e'

$$F_W = \frac{ee'}{r^2}\left\{1 - \left(\frac{\dot{r}}{c}\right)^2 + 2\frac{r\ddot{r}}{c^2}\right\}. \tag{1.14}$$

Weber's force (1.14) takes into account the relative motion of two charges and their orientation but not their angular dependence, and their limiting speed, c. The angular dependence can be introduced through $\dot{r}^2 \to \dot{r}^2 + r^2\dot{\theta}^2$ in which case (1.14) becomes

$$F_W = \frac{ee'}{c^2 r}\left\{\frac{c^2 - \dot{r}^2}{r} - r\dot{\theta}^2 + 2r\ddot{r}\right\}. \tag{1.15}$$

Whereas the first term in (1.15) is related to the contraction in the direction of the motion, the last two terms are analogous to Newton's law, accounting for radial acceleration and angular momentum. Newton's law is purely mechanical, and there is nothing that would indicate a limiting speed.

In regard to Weber's law, (1.14), the terms involving Ampère's law contain c^{-2}. Ampère established that the ratio between parallel cur-

[2] J.M.A. Danby, *Fundamentals of Celestial Mechanics*, Willmann-Bell Inc., 2nd ed., 1988, Sec 4.9

rent elements to the force between longitudinal current elements were in the ratio $1:2$.

In Weber's *Sixth Memoir* published in 1871, there appears a critical length associated with the reversal of the Coulomb force. In his words: [3]

> when particles e and e' are of the same kind, *they do not always repel each other*; thus when $\dot{r}^2 < c^2 + 2r\ddot{r}$, they repel so long as
>
> $$r > \frac{2ee'}{mc^2},$$
>
> and, on the contrary, they attract when

the inequality is reversed. The mass, m, appears if, for no other reason, than the sake of dimensions. The right-hand side will be recognized as twice the classical electron radius. This was no small feat taking into account that the year was 1871.

In fact, John Michell, a letter to his friend Henry Cavendish in 1783, predicted that once a star exceeded that of the Sun in proportion $500:1$, he prophesied that

> supposing light to be attracted by the same force in proportion to its *vis inertiae*, with other bodies, all light emitted from such a body would be made to return towards it by its own proper gravity This assumes that light is influenced by gravity in the same way as massive ob-

[3] L Hecht, "The significance of the 1845 Gauss-Weber correspondence," *21st Century* (1996) 22-43.

jects.

1.2 From Euclidean to non-Euclidean planes

Gauss assumed that any two dimensional surface in a three-dimensional world had, at least locally, a geodesic polar parameterization, (r, θ). Euclidean straight lines are also geodesics with a flat metric

$$ds^2 = dr^2 + r^2 d\theta^2, \tag{1.16}$$

which gives rise to Newton's slope, (1.7). Our interest, however, is closed trajectories, of which the ellipse (1.11) is an example. In order to go beyond the Euclidean plane, we might try hyperbolic and elliptic planes of constant curvature whose respective metrics are given by

$$ds^2 = dr^2 + R^2 \sinh^2\left(\frac{r}{R}\right) d\theta^2, \tag{1.17}$$

$$ds^2 = dr^2 + R^2 \sin^2\left(\frac{r}{R}\right) d\theta^2, \tag{1.18}$$

where R is a characteristic scale factor in non-Euclidean geometries.

Unlike the elliptic plane, where all trajectories are closed, the hyperbolic plane admits both closed and open trajectories. According to (1.17), Newton's slope will be given by

$$\frac{1}{R\sinh(r/R)}\frac{dr}{d\theta} = \pm\tan\theta. \tag{1.19}$$

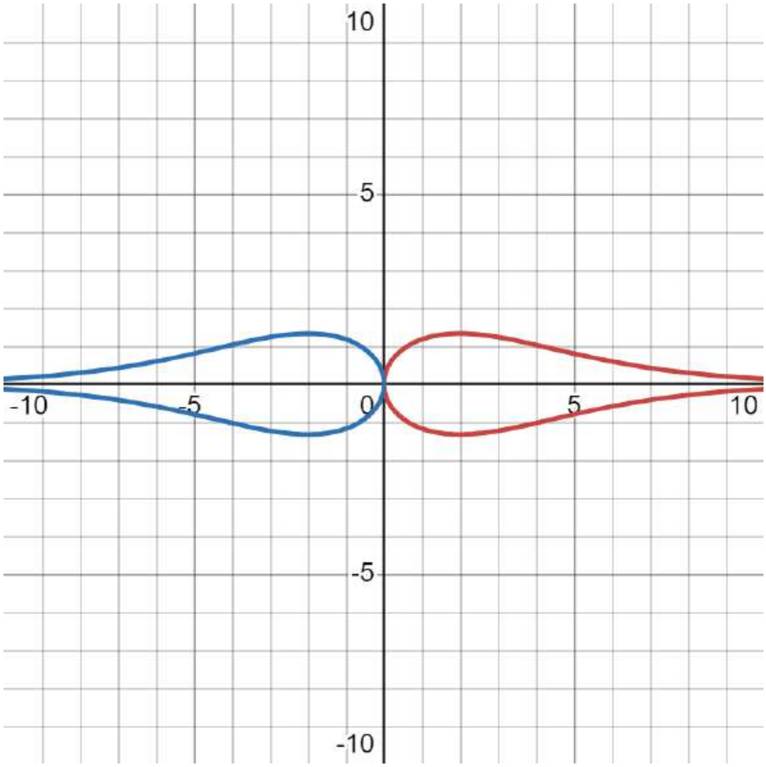

Figure 1.2: Two symmetric Joukowski ellipses in the hyperbolic plane of constant, negative curvature.

Choosing the minus sign, and integrating gives

$$\tanh\left(\frac{r}{2R}\right) = \coth\left(\frac{r}{R}\right) - \mathrm{csch}\left(\frac{r}{R}\right) = \cos\theta, \tag{1.20}$$

where, for simplicity, we have suppressed the arbitrary constant of integration. Solving for r results in

$$r = \pm 2R\tanh^{-1}(\cos\theta) \tag{1.21}$$

which are two Joukowski ellipses shown in (1.2).

Taking the derivative of (1.21), we find $r' = 2\csc\theta$, and whose integral is

$$\theta = 2\tan^{-1}\left(\exp\left(\frac{r}{2R}\right)\right) \tag{1.22}$$

where R is a constant of integration. Equation (1.22) is just another way of writing the Joukowski ellipse.

From the metric in the hyperbolic plane of constant negative curvature where $G = \sinh^2(r)$, we obtain the equation for Newton's slope as [cf. Eqn (1.19):

$$\frac{1}{\sinh(r)}\frac{dr}{d\theta} = \pm\tan\theta.$$

Now choosing the positive sign, we have the integral:

$$r = -2\tanh^{-1}(\cos\theta) = \ln\left(\frac{1-\cos\theta}{1+\cos\theta}\right).$$

This agrees with (1.22) when we write it in the form

$$r = \ln\tan\left(\frac{\theta}{2}\right) = \ln\left(\frac{1-\cos\theta}{1+\cos\theta}\right).$$

In the hyperbolic plane, the inverse of the radial coordinate, $u = \coth(r)$, and $\sqrt{u^2-1} = \mathrm{csch}(r)$. Noting that the secant is

$$u + \sqrt{u^2-1} = \sec\theta, \tag{1.23}$$

we can add it to (1.20) to get

$$u = \frac{1}{2}(\sec\theta + \cos\theta). \tag{1.24}$$

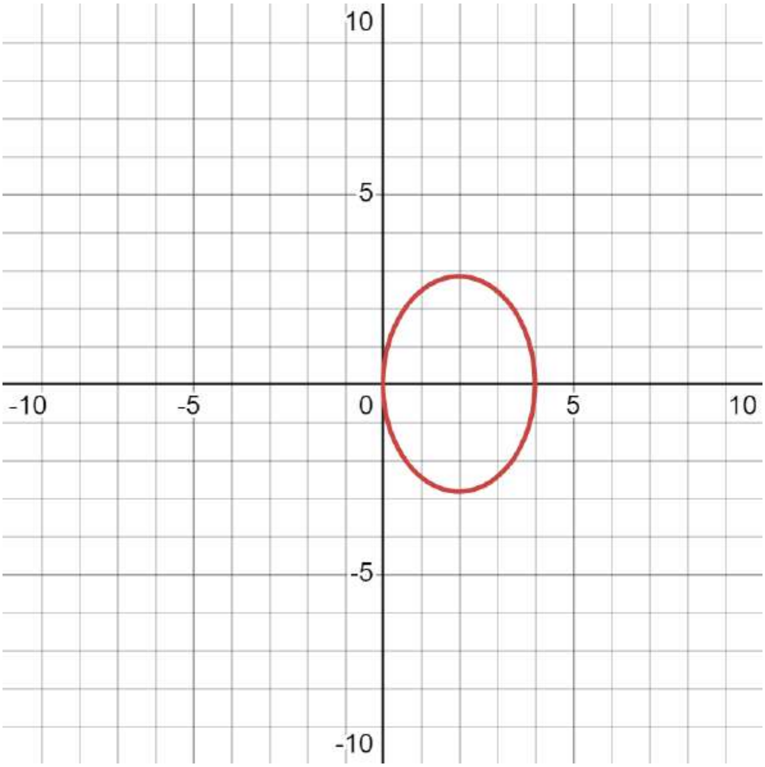

Figure 1.3: The reduction to the Euclidean plane gives an ellipse through the origin.

Equation (1.24) transforms in an analogous way to the one that stretches a circle to an ellipse in the complex plane. In the Euclidean plane, where $u = 1/r$, (1.24) gives an ellipse

$$1/r = \frac{1}{2}(\cos\theta + \sec\theta), \tag{1.25}$$

passing through the origin, as shown in (1.3). Unlike the conventional expression for a conic, containing two arbitrary constants as in (1.11), (1.25) does not contain any constant whatsoever. It is analogous to a circle passing through the origin that is caused by an inverse-fifth force, as we shall now discuss.

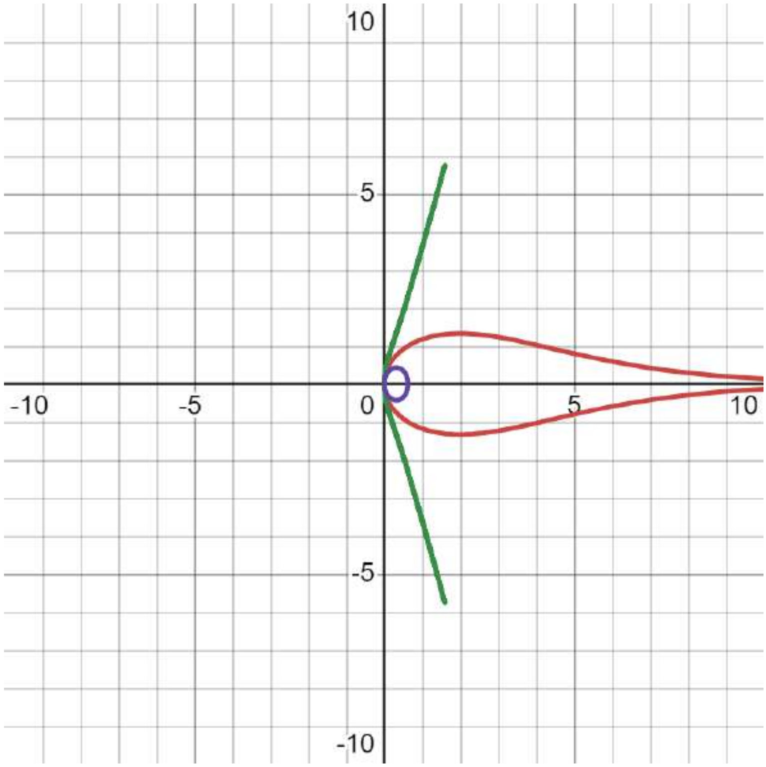

Figure 1.4: The Joukowski ellipse, $A = 1/2$, with its characteristic cusp, characteristic of airfoils, separates open, $A < 1/2$, from closed, $A > 1/2$, trajectories. As $A \to 1$, the ovals transform into ellipses passing through the origin.

The ellipse in the Euclidean plane, (1.25) becomes a Joukowski ellipse,

$$r = \coth^{-1}(A(\cos\theta + \sec\theta)) \tag{1.26}$$

in the hyperbolic plane of constant curvature for the value $A = 1/2$, as shown in (1.4).

Having noticed the similarity between an ellipse through the origin and a circle, we write down the Binet equation

$$u'' + u = \sec^3\theta = (u + \sqrt{u^2 - 1})^3. \tag{1.27}$$

Let us consider the terms in the expression for the secant,

$$\sec\theta = u + \sqrt{u^2 - 1} = \frac{1}{\tanh(r)} + \frac{1}{\sinh(r)}. \tag{1.28}$$

Since the Lobachevskian 'straight' line is $\tanh(r)$, and this gives the Newtonian potential as

$$V = \frac{\mu}{\tanh(r)},$$

where $\mu = GM$ is the gravitational potential. Consequently, the force is

$$F = \frac{dV}{dr} = -\frac{\mu}{\sinh^2(r)}. \tag{1.29}$$

Thus, for large values of r, the secant will be dominated by the first factor in (1.28), and the force will be essentially and inverse-fifth power of the radial coordinate. This gives a circle through the origin. Consequently, the second term, which is proportional to $\sinh(r)$ in the denominator of the force, introduces distortions in the circle transforming it into an oval or an ellipse.

1.3 Newton's impact method

The proof of the expression for the centrifugal force in the hyper-

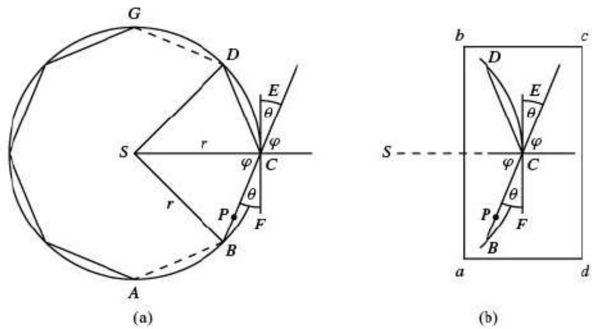

Figure 1.5: A particle at P will collide with a hyperbolic circle at point C. The image is taken from R L Lamphere, "Solving the non-euclidean uniform circular motion problem by Newton's impact method", *Math Mag* **83** (2010), p. 366.

bolic plane,

$$CF = \frac{v^2}{\sinh(r)} \tag{1.30}$$

can be accomplished by Newton's impact method. [4] There is a serious error in quoted papers that the editors refused to correct claiming that the paper was 'too old' for such a correction to be of any interest to their readers. Errors are never too old to correct! The centrifugal force is given there as $v^2/\tanh(r)$ instead of (1.30).

The circular path is discretized by an n-sided regular polygon. The polygonal path is inscribed in a circle of radius r with center S, as shown in (1.5).

Keeping in mind that we are dealing with uniform circular motion, like Newton did. The only difference is that we are going to replace the distance in the source by the hyperbolic distance and not by the radius of curvature, as Newton did.

[4] R L Lamphere, "Solving the non-euclidean uniform circular motion problem by Newton's impact method", *Math Mag* **83** (2010) 366.

Moreover, there is the problem that you cannot localize a body smaller than its wavelength. The LIGO interpretation of a GW travelling down to earth and stretching space along one arm while compressing it along the other arm at periodic intervals is untenable. The passing GW will 'see' a speck, and certainly that 'speck' cannot make out the properties of the wave like its polarization.

At any given instant, so LIGO claims, more space along one arm and less in the other will cause the laser beam to travel different distance so that they will recombine in different phases. The resulting brightness is used to produce a current that returns the test masses back to their original positions. However, the minute measurements of the displacement of the mirrors would make the their momenta completely unknown make it difficult for the current, or any other means, to return them to their initial positions.

Recognizing the problem at hand, Thorne writes in the forward to *Quantum Measurement* [68] belittling the textbooks on quantum mechanic written during the period $1940 - 1970$. by saying that they have "little respect or interest in the quantum theory of measurement." Surely, he has not read David Bohm's *Quantum Theory* published in 1949. There, he spells out in detail the quantum theory of measurement, and why Heisenberg's uncertainty principle has to be reckoned with in *all* quantum measurements.

After admitting that the act of "measurement has produced an irreversible and indeterminate change in the quantum object," the authors then go on to define a "non-demolition" quantum measurement. Their motivation was:

[68] V B Braginsky & F Ya Khalili, *Quantum Measuremet* K S Thorne, ed. (Cambridge U P, Cambridge, 1992).

> In the 1970s, in connection with efforts to construct detectors for gravitational waves, it became necessary to invent methods for measuring macroscopic observables at levels of precision approaching and exceeding the standard quantum limits However, theoretical analyses of typical, traditional schemes of measurement showed that their precisions can never exceed the quantum limit, even in principle. The solution to this dilemma, it was recognized, was to use a nontraditional class of measurement schemes, carefully crafted to overcome the standard quantum limits. For these schemes was coined the term 'quantum non-demolition [QND] measurements.'

Their panacea is to take "long enough for the measurements [so] one can obtain any desired sensitivity." For instance, the minimum error in the measurement of energy is of the order $\hbar/\tau$. All that is necessary is to let time $\tau \to \infty$. This then defines a vacuous steady state in which the energy can be measured with unlimited precision but absolutely nothing can be said about the time needed to perform the measurement. And all types of unintentional interactions have certainly occurred within that time period.

They then go one to illustrate how such a QND measurement can be performed. Radiation pressure is considered in which it is necessary to consider an extremely small pressure, even at optical frequencies. To obtain such weak pressures it is necessary to "register the pressure produced by a few quanta."
However, the fluctuations will be so great that one cannot measure such a radiation pressure. Try measuring the "pressure" attributed to a single particle in a box! It is a well-known thermodynamic consequence that below a certain level, usually taken as as the va-

lidity of Stirling's approximation, that macroscopic measurements cannot be performed on such systems. [69]

[69] B H Lavenda, *Statistical Physics: A Probabilistic Approach* (Dover, New York 2015).

Epilogue

If anything, the discovery of GWs should be more alarming than reassuring. It is a clear case where the outcome was determined before the experiment was done! Yet, GR is certainly not the first theory to have predicted them.

Maxwell gave up trying to extend his field theory of EM to gravitation. For where do you find a situation where, when matter is widely separated, the forces are the least while the potential energy is greatest, whereas when the potential energy is least when the forces are greatest? This meant to Maxwell that he was dealing with a field with *negative* energy — something he found so repugnant that he gave the matter up entirely. But that did not carry over to his protégé, Oliver Heaviside.

Though his disciple, Oliver Heaviside, [70] did not, and went ahead to derive a theory of gravitation that was based on the non-instantaneous propagation of the gravitational force. At the end of his expose he admitted that it

> does not enlighten us in the least about the ultimate nature of gravitational energy. It serves, in fact to further illustrate the mystery. For it must be confessed that the exhaustion of potential energy from a universal medium is a very unintelligible and mysterious matter.

Heaviside modelled the gravitational acceleration $\vec{g}$, after an electric field. If it is the gradient of a scalar potential there is no further

[70] O Heaviside, "A gravitational and electromagnetic analogy," in *Electromagnetic Theory* Vol. I (The Electrician, London, 1898) pp 455-466.

ado. But, citing Newton‘s letter to Bentley, he said that “it is as incredible now as it was in Newton’s time that gravitative influence can be exerted without a medium,” and that in that medium it propagates at a finite speed, v.

Heaviside considers $\vec{g} = -\nabla\phi$ as the condition that “the gravitational force is *exactly* dependent on the configuration of the matter.” This he takes equivalent to $\nabla \wedge \vec{g} = 0$. So when this is not so the last relation must be invalid. And if it is invalidated, it should be replaced by:

$$cv^2 \nabla \wedge \vec{g} = \dot{\vec{h}}, \tag{5.49}$$

which introduces an *auxiliary* field, $\vec{h}$, where c is a constant.

If the auxiliary field, $\vec{g}$, is divergence free, its curl must be non-zero,

$$\nabla \wedge \vec{h} = -c\dot{g}, \tag{5.50}$$

whose source term Heaviside modeled after Maxwell‘s displacement current, $-c\dot{\vec{g}}$. Then taking the curl of (5.49), and the time derivative (5.50), the auxiliary field can be eliminated to obtain:

$$-v^2 \nabla \wedge (\nabla \wedge \vec{g}) = \ddot{\vec{g}}. \tag{5.51}$$

which is Heaviside‘s *transverse* wave equation since:

$$\nabla^2 = \nabla\text{div} - \text{curl}^2. \tag{5.52}$$

The vector identity (5.52) implies $\nabla \cdot \vec{g} = 0$. This motivates considering the analogy with EM where $\vec{g}$ would be analogous to an electric field:

$$\vec{g} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}, \tag{5.53}$$

which represents an "oscillating gravitational radiation field.[71] It should also

> be transverse to the wave propagation direction and to have an amplitude that falls off as $1/r$, the usual space dependence for the amplitude of [EM!] waves far from their sources.

Consequently, only the second term in (5.53) subsists so that "the gravitational radiation field $\vec{g}$ depends only on the components of the vector potential $\vec{A}$ that are transverse to the observation direction:

$$\vec{g} = -\frac{\partial \vec{A}}{\partial t}. \tag{5.54}$$

So that if $\nabla \cdot \vec{g}$, so, too, will $\nabla \cdot \vec{A} = 0$, which is referred to as the Coulomb gauge.

Is the analogy with EM waves relevant? EM waves are the only known waves that do not need a material medium to propagate in. Some media can support both transverse and longitudinal waves, such as ocean waves. Now if GWs are *mechanical* transverse waves they need a material medium, and propagate by means of vibrations that are perpendicular to the direction of propagation: the so-called 'ripples' of spacetime. This is like some sort of jello which robs the vacuum of its vacuous state. We have come full circuit by re-introducing the ether that Einstein say fit in 1905, but later

[71] R C Hilbron, "Gravitational waves from rotating binaries without general relativity" a tutorial, 09/2017.

repented having done so in 1920.

It is the auxiliary field, $\vec{h}$, that allows EM waves to propagate in "vacuum", and this has been borrowed by GR, and named the gravitomagnetic field. If it exists then the ripples on spacetime are completely superfluous so that GWs can propagate throughout the universe just like EM waves, and since they travel at the same speed, this would be the more logical choice. The two orthogonal vector fields ride piggy-back, and this is what allows the EM waves to propagate in a vacuum. The presence of a material medium tends to slow down their propagation by offering resistance to their propagation.

However, if they do need a material medium to propagate in, the polarizations of GWs must be characteristic of that medium rather than an intrinsic property of the waves themselves. And because they are considered as undulations in spacetime, they must show optical diffraction phenomena and aberration. Heaviside, too, was perplexed:

> The remarks of the Editor and of Prof Lodge on gravitational aberration, lead me to point out now some of the consequences of the modified law when we assume that the ether is the working agent in gravitational effects, and that it propagates at speed v.

Heaviside considers how the gravitational force between the Sun and the Earth, f, is modified "when the Sun is in motion at speed

u through the ether. The modified force law,

$$F = f \times \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}, \tag{5.55}$$

where $\beta = u/v$ is the relative velocity and θ is the angle between the line of motion and the radius vector between the Sun and the Earth. Except for the error that the term in the numerator $1-\beta^2$ of (5.55) should be squared, the expression is identical to the result found by Ibison *et. al.* [72] found in §4.6, and pre-dates the latter by more than a century

Moreover, like IPL and Carlip, [73] Heaviside can't answer the objections of Lodge because (5.55) is a radial, and not a tangential, force.

What more can GR add? Eddington [74] addressed the problem, referring to two earlier papers by Einstein who investigated the propagation of GWs with the speed of light "due to changes in the distribution of matter," not mentioning that only accelerating masses are able create "the altered curvature of space-time," that resulted from gravitational radiation

Since the theory is tensorial, it admits three types of GWs: longitudinal-longitudinal, longitudinal-transverse, and transverse-transverse. [75] Eddington shows that only the latter "are propagated with the speed of light *in all systems of co-ordinates*." Einstein found that the first two categories of waves "convey no energy." This seems a little strange since GR cannot localize energy, and it

[72] M Ibison, H E Puthoff & S R Little, "The speed of gravity revisited."

[73] S Carlip, "Aberration and the speed of gravity," arXiv:gr-qc/9909087v2.

[74] A S Eddington, "The propagation of gravitational waves," *Proc Roy Soc London* Series A (1922) 268-282.

[75] H Weyl, *Space Time Matter* (Dover, New York, 1952) p 252.

explains why Einstein performed instead a classical calculation of a spinning rod losing energy by the emission of GWs that made use of the Poynting vector, which we discussed in §5.5.2. This explicitly involves the existence of an auxiliary field, $\vec{h}$, since Poynting's vector is:

$$\vec{S} = \frac{v}{4\pi}(\vec{g} \wedge \vec{h}).$$

The facts that "plane waves are a very special, and artificial case of gravitational wave propagation," lead Eddington to consider *divergent* waves. And

> although the equations of the theory are the same as those occurring in the propagation of sound waves, there is no propagation of gravitational waves uniformly in all directions like a spherical sound wave.

However, we should not expect sound waves to be uniformally propagated in all directions since they are *longitudinal* with material motion in the direction of propagation.

To consider this possibility further, suppose that the gravitational field is the negative gradient of the *velocity* potential, ϕ, i.e.,

$$\vec{g} = -\nabla\phi.$$

Let $P = \dot{\phi}$ be the power loss due to the emission of GWs, where the rate of power loss is given by:

$$\dot{P} + v^2\nabla \cdot \vec{g} = 0.$$

Replacing P and $\vec{g}$ by their definitions in terms of the velocity potential, ϕ, leads immediately to the scalar wave equation,

$$\ddot{\phi} - v^2\nabla^2\phi = 0.$$

These are longitudinal waves that propagate at a finite speed v, whatever that may turn out to be. The bad news is that they are non-polarized, but this is more than compensated by the fact that you do not have to explain gravitational aberration, and have the possibility of impacting matter just like pressure waves. Albeit, this is skirting entirely the basic question of what is causing the polarizing of the GWs.

Each model has its own attributes, and short-comings. The analogy with EM waves necessarily attribute to GWs the phenomena of diffraction and aberration. EM waves can be shielded whereas GWs cannot. Rather, if GWs are mechanical transverse waves, they need a material medium, and propagate by means of vibrations of the medium normal to the direction of propagation. The type of polarization would then be characteristic of the material medium, whereas for EM waves, the polarizations are given by the Grassmann force, and are equal and opposite in directions normal to the motion.

Yet, none of these models would explain the lack of shielding of gravity, and what we are left with is a compromise: The optical properties of gravity.

Index

Bernard Lavenda

Gravitational Optics in Curved Spacetimes: Contrasting Newtonian and Relativistic Perspectives

This book offers a critical appraisal of where we stand on a formulation of a theory of gravitation. Newtonian theory is reformulated in the general form of conics and their relation to aberrancy. Conics are to planetary orbits what Cassini ovals are to binaries. The transition from a Bernoulli lemniscate to Cassini ovals is discussed in terms the instability criteria of Roche lobes, and their fission. When the second derivative of the curve is no longer constant, the Newton inverse law does not apply and new central forces appear which can be derived from the radius of aberrancy, just as the inverse-square law follows from the radius of curvature. The logarithmic spiral becomes periodic in velocity space, and a new Hubble law appears between acceleration and velocity. This has the effect of decreasing the decay rate of accelerations so that they can explain the flat rotational curvatures oberserved in spiral galaxies. Numerical relativity has reduced its conception of general relativity to a Le Sage- type theory thereby making it refutable.

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